WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2019
TEST 1: Complex numbers

Name: ________________________

Thursday 7th March

Time: 25 minutes

Total marks: $\frac{\quad}{25} + \frac{\quad}{30} = \frac{\quad}{55}$

Calculator free section

1. [6 marks – 2 each]

a) Convert each of $1+\sqrt{3}\,i$ and $\sqrt{3}-i$ to polar (cis) form.

b) Let $\omega = \dfrac{\left(1+\sqrt{3}\,i\right)^6}{\left(\sqrt{3}-i\right)^k}$. Show that $\omega = 2^{6-k}\,\text{cis}\left(\dfrac{k\pi}{6}\right)$.

c) For which values of k is ω purely imaginary? $(-\pi < \arg(\omega) \le \pi)$

2. [7 marks – 3 and 4]

$z = a + bi$ represents a complex number, with *a* and *b* both real numbers.

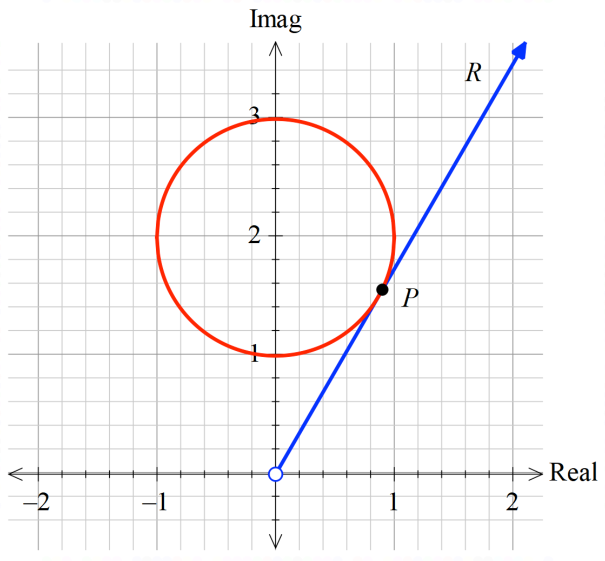
a) Evaluate *a* and *b* if $2z + iz = 4 - 3i$

b) Develop an equation relating *a* and *b* if $\mathrm{Re}\left(\frac{\bar{z}+1}{z}\right) = 1$

3. [6 marks – 1 each]

The unit circle shown has centre $(0, 2)$ and the ray *R* is a tangent at point *P*.

The circle represents a locus of complex numbers *z* and *P* is the complex number ω.



Determine:

(a) an equation for the circle, in terms of *z*

(b) $|\omega|$

(c) $\arg(\omega)$

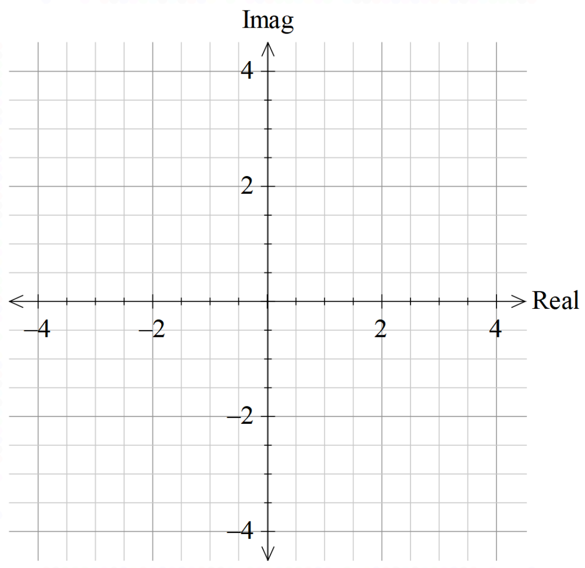
(d) ω expressed in Cartesian form $a + bi$

(e) an equation for *R*, in the form $\text{Im}(z) = m \times \text{Re}(z) + c$, for $\text{Re}(z) > 0$

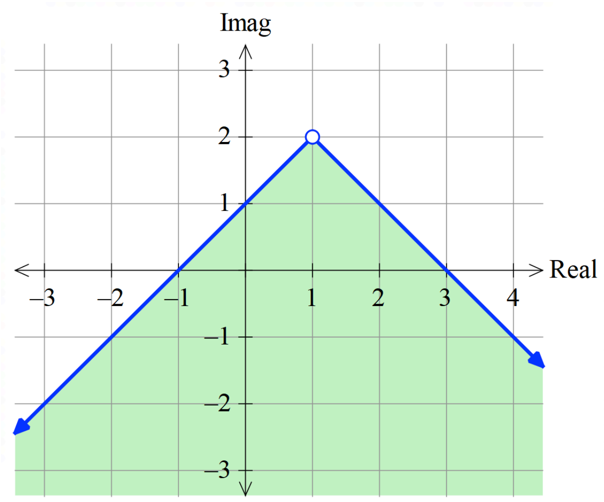
(f) the maximum value of $\arg(z)$ for the circle.

4. [6 marks – 3 each]

(a) Sketch, on the axes provided, the region which satisfies $\dfrac{|z+2i|}{|z+1-i|} > 1$



(b) Use inequalities, involving the argument of a complex number, to describe the shaded region:



Test 1: Complex numbers

Name: ________________________

Time: 30 minutes 30 marks

Calculator assumed section

5. [9 marks – 1, 2, 3 and 3]

Let $P(z)$ be a cubic polynomial with real co-efficients. It can be written as the product of a linear factor and a quadratic factor; i.e. $P(z) = (az + b)(z^2 + cz + d)$ with *a*, *b*, *c* and *d* all real.

(a) $z = 2 - i$ is a solution to $P(z) = 0$. Write down another solution.

(b) Hence evaluate *c* and *d*.

When $P(z)$ is divided by $(z - 1)$ the remainder is 6 and when $P(z)$ is divided by $(z - 2)$ the remainder is 5.

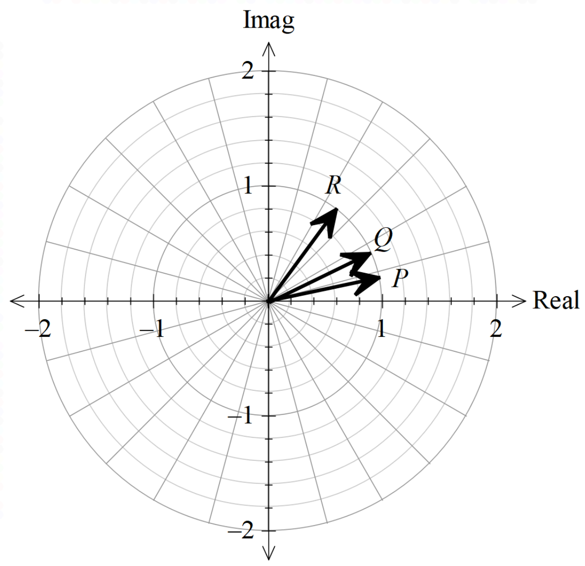
(c) Evaluate *a* and *b*.

(d) Write $P(z)$ in expanded form (free of brackets) and hence, or otherwise, list all the zeroes of $P(z)$.

6. [8 marks – 2, 3, 2 and 1]

On this Argand diagram, P represents the complex number $z = \text{cis}\,\theta$, for $0 \le \theta \le \dfrac{\pi}{2}$.

Q and R represent z^2 and z^4.



a) Add the complex number $z^2 + z^4$ to the diagram

b) Use the geometry of the situation, or otherwise, to show that $\arg\left(z^2 + z^4\right) = 3\theta$

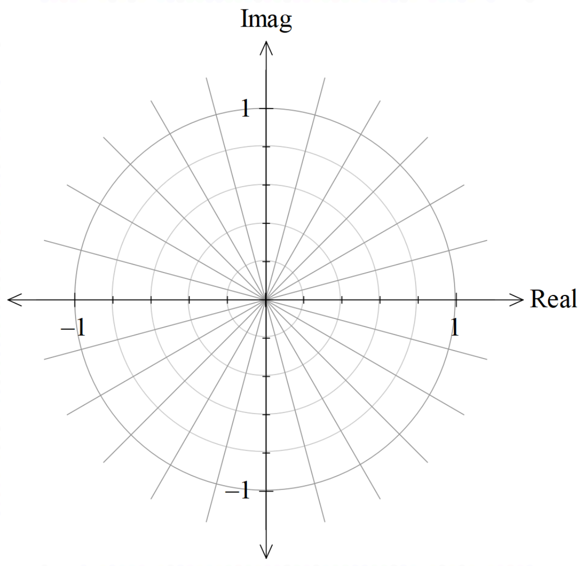
c) Prove that $\left|z^2 + z^4\right| = 2\cos\theta$

d) Explain why $z^2 + z^4 = 2\cos\theta\,\text{cis}\,3\theta$

7. [13 marks – 4, 1, 2, 1, 2 and 3]

a) List all the solutions to $z^5 + 1 = 0$ for $-\pi < \arg(z) \le \pi$

b) Represent these solutions as z_1 to z_5 on the Argand diagram, with z_1 in the first quadrant and z_5 in the fourth.



c) Show that $|z_1 - z_5| = 2\sin\dfrac{\pi}{5}$

d) Determine an expression for the perimeter of the pentagon formed by joining the solutions to $z^5 + 1 = 0$

See over for parts e and f

e) Verify that the area of the pentagon is $\frac{5}{2}\sin\left(\frac{2\pi}{5}\right)$

f) Generalise: determine the perimeter and area of the polygon formed by the solutions to $z^n+1=0$. What happens as $n\to\infty$?